

Bidding Price Game Model

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Abstract—There are two main quotation mode in current bidding, which are without base price and compound base price. Bidders should bid accurately and reasonably to win a bid. Based on game theory and the characteristics in bidding, this paper constructed bidding price game model of without base price and bidding price game model of compound base price to provide evidence for bidding decision. Relevant factors of optimal bidding were found by analysing models results. The probability of bid winning can be improved by analysing these factors effectively. At last, the methods of preventing string bid phenomenon were proposed by comparative analysis of these two models.

Keywords- bidding price , without base price, compound base price, game

I. INTRODUCTION

With Law on Public Bidding is promulgated, bid winning is the fundamental way to obtain construction project. In order to bid successfully and selectively in current competitive bid market, contractors need to make decisions objectively, including whether bid or not and how to bid. Bidding price is an important problem of bidding decision. Without base price and compound base price are two main bidding forms. Game theory[1] can apply to bid decision according to theoretical and practical analysis. Therefore, bidding price game model of without base price and bidding price game model of compound base price are constructed and compared. Then this paper finds significant factors in bidding by analysing model results, thus, the probability of bid winning can be improved.

II. THE CONCEPT OF WITHOUT BASE PRICE AND COMPOUND BASE PRICE

A. The concept of without base price

Without base price means that the tenderer does not compile base price which is decided by valid quotation average. The contractor can win a bid when his bidding price is close to base price or the lowest bidding price evaluated by the tenderer[2]. Law on Public Bidding stipulated that the lowest bidding price can not be lower than

actual cost. We can interpret that the lowest bidding price contains actual cost and meager profit.

B. The concept of compound base price

There are two parts in compound base price[3], which are base price compiled by the tenderer and valid quotation average. And each part has some weight. The basic formula is $C = k_1 \times A + k_2 \times B$, let C be compound base price, let A be the base price compiled by the tenderer, let B be valid quotation average, let k_1 be the weight of A , let k_2 be the weight of B , and $k_1 + k_2 = 1$. The formula of B is

$$B = \frac{1}{n} \sum_{i=1}^n B_i, \quad B_i (i=1,2,\dots,n) \text{ is bidder } i \text{ ' valid quotation,}$$

and n is the number of bidders. There are three parts in bidding documents, which are business credit, construction organization technology, and bidding price. And these three parts decide bid winning together. The business bidding document and technology bidding document are not explained in this paper, for they are manifestations of contractor's management ability and easy to grip and control in compiling bidding documents.

III. BIDDING PRICE GAME MODEL OF WITHOUT BASE PRICE

A. Basic hypothesis

Assumption 1. The base price is B . ($B = \frac{1}{n} \sum_{i=1}^n B_i$,

$B_i (i=1,2,\dots,n)$ is valid quotation of bidder i , n is the number of bidders whose bidding price is valid. This formula is known from the definition of without bidding price. Bid winning price is the price which is close to base price B .)

Assumption 2. The bidder i only know the cost C_i , other bidders do not know the cost C_i , and the bidder i does not know the cost $C_j (j=1,2,\dots,n, \& j \neq i)$. (Bidders can figure out comprehensive cost according to requirements of bidding documents, project properties, scope, technical

specification, schedule requirements, construction scheme to be used, progress plan, labor budget price, materials budget price, and machinery budget price before making the bidding price. Because construction scheme and progress plan of each construction unit are different, the bidder $i(i=1,2,\dots,n)$ does not know the cost of the bidder $j(j=1,2,\dots,n,&j \neq i)$, and so does the bidder j .)

Assumption 3. The bidding price of the bidder i is B_i , and $B_i = k_i \cdot C_i$ (Generally speaking, the bidding price is made up of adjusted engineering cost and expected profit margin. Construction industry in our country implements meager profit policy. However, in order to encourage competition, construction units can take flexible profit margin appropriately in actual bidding.)

Assumption 4. The bidder i knows C_i is a uniform distribution function defined in $[L, H]$ interval. At the same time, bidder j also knows that C_i of bidder i is a uniform distribution function defined in $[L, H]$ interval, that is, the cost distribution function $f(x)$ is public information. (According to enterprise quota and engineering quantity, the bidder can calculate the cost of his company. The situation of the enterprise decides strategies, including low-cost strategy and high-cost strategy. Bidders can estimate the cost scope of competitors according to former bidding. The accuracy of cost scope is decided by the accuracy of mastering other bidders' information. It is assumed that X obeys uniform distribution in $[a, b]$ interval when we can not distinguish the difference of the possibility which X takes different values in $[a, b]$ interval. In order to deal with incomplete information, Harsanyi transformation was introduced. Payoff of a player changes to a problem of probability, so we introduce uniform distribution to explain that the probability is only related to interval length. In other words, the probability of winning is relevant to mastered information. The more information you master, the more accuracy of competitors' bidding price you estimate, then the probability of bid winning will be increased. Hence, this assumption coincides with the actual situation.)

B. Two-person bidding price game model of without base price

$$U_i(B_i, B_j, C_i, C_j) = \begin{cases} B_i - C_i & B_i < B_j \\ \frac{B_i - C_i}{2} & B_i = B_j \\ 0 & B_i > B_j \end{cases} \quad (1)$$

and $i, j = 1, 2, & i \neq j$.

$$\text{So } n = 2, \quad B = \frac{B_1 + B_2}{2}.$$

The problem faced by the bidder i is how to achieve profit maximization, that is

$$\max \left\{ (B_i - C_i) \cdot \text{prob}(B_i < B_j) + \frac{B_i - C_i}{2} \cdot \text{prob}(B_i = B_j) + 0 \right\}$$

, $\text{prob}(B_i < B_j)$ represents the probability that the bidding price of bidder i is less than the bidding price of bidder j . Because $\text{prob}(B_i = B_j) = 0$

So

$$\begin{aligned} & \max \left\{ (B_i - C_i) \cdot \text{prob}(B_i < B_j) + \frac{B_i - C_i}{2} \cdot \text{prob}(B_i = B_j) + 0 \right\} \\ & = \max \left\{ (B_i - C_i) \cdot \text{prob}(B_i < B_j) \right\} \\ & = \max \left\{ (B_i - C_i) \cdot \text{prob}(B_i < B_j < B) \right\} \\ & = \max \left\{ (B_i - C_i) \cdot \text{prob}(B_i < k_j C_j < B) \right\} \\ & = \max \left\{ (B_i - C_i) \cdot \text{prob} \left(\frac{B_i}{k_j} < C_j < \frac{B}{k_j} \right) \right\} \end{aligned}$$

Because C_j is a uniform distribution function defined in $[L, H]$ interval.

$$\begin{aligned} \text{So } \text{prob} \left(\frac{B_i}{k_j} < C_j < \frac{B}{k_j} \right) &= \frac{\frac{B}{k_j} - \frac{B_i}{k_j}}{H - L} \\ &= \frac{B - B_i}{k_j(H - L)} \end{aligned}$$

$$\text{So } \max \left\{ (B_i - C_i) \cdot \text{prob}(B_i < B_j < B) \right\}$$

$$= \max \left\{ (B_i - C_i) \cdot \frac{B - B_i}{k_j(H - L)} \right\}$$

$$= \max \frac{B_i B - B_i^2 - C_i B + C_i B_i}{k_j(H - L)}$$

Seeking first-order partial derivative of B_i

$$B - 2B_i + C_i = 0$$

$$\text{So } B_i^* = \frac{B + C_i}{2} = \frac{1}{2}(B_1 + B_2) + C_i$$

$$\text{That is } B_1^* = \frac{B_1 + B_2}{4} + \frac{1}{2}C_1 \quad \text{and} \quad B_2^* = \frac{B_1 + B_2}{4} + \frac{1}{2}C_2.$$

The optimal bidding of bidder i is not only related to his own cost and bidding price, but also related to bidding price of competitors according to model results. Thus, bidders can improve bidding level by analysing competitors' useful information.

C. Multi-person bidding price game model of without base price

$$U_i(B_i, B_j, C_i, C_j) = \begin{cases} B_i - C_i & B_i < B_j \\ B_i - C_i & B_i = B_j \\ 0 & B_i > B_j \end{cases} \quad (2)$$

and $i, j = 1, 2, \dots, n$ & $i \neq j$.

$B = \frac{1}{n} \sum_{i=1}^n B_i$, B_i ($i = 1, 2, \dots, n$) is valid quotation of

bidder i , n is the number of bidders whose bidding price is valid. The payoff of bidder i is

$$U_i = \max \left\{ (B_i - C_i) \prod_{j=1, i \neq j}^n \text{prob}(B_i < B_j) \right\}. \text{ Based on}$$

two-person bidding price game model, the followings are derived.

$$U_i = \max \left\{ (B_i - C_i) \prod_{j=1, i \neq j}^n \text{prob}(B_i < B_j) \right\}$$

$$= \max \left\{ (B_i - C_i) \frac{\left(\frac{B - B_i}{k_j(H - L)} \right)^{n-1}}{\prod k_j^{n-1}} \right\}$$

Seeking first-order partial derivative of B_i

$$\frac{\partial U_i}{\partial B_i} = (B - B_i)^{n-1} - (B_i - C_i) \cdot (n-1) \cdot (B - B_i)^{n-2}$$

Let $\frac{\partial U_i}{\partial B_i}$ be zero

$$\text{So } (B - B_i)^{n-1} - (B_i - C_i) \cdot (n-1) \cdot (B - B_i)^{n-2} = 0$$

$$B_i = \frac{B + C_i \cdot (n-1)}{n}$$

$$\text{So } B_i^* = \frac{1}{n^2} \sum_{i=1}^n B_i + \frac{n-1}{n} C_i$$

The conclusion is consistent with the conclusion of two-person bidding price game model of without base price when n is two. Thus, the optimal bidding of bidder i is not only related to his own cost and bidding price, but also related to bidding price of competitors and the number of bidders.

IV. BIDDING PRICE GAME MODEL OF COMPOUND BASE PRICE

A. Basic hypothesis

These assumptions coincide with assumptions in without base price, but assumption 1 is different. Assumption 1. The base price is B . ($B = k_x \frac{1}{n} \sum_{i=1}^n B_i + k_y D$, this formula is known from the definition of compound base price.)

B. Multi-person bidding price game model of compound base price

The modeling process coincides with the without base price modeling process. Hence, the process is ignored.

$$\text{The optimal bidding is } B_i = \frac{B + C_i \cdot (n-1)}{n}.$$

$$\text{So } B^* = \frac{k_x \frac{1}{n} \sum_{i=1}^n B_i + k_y D + C_i(n-1)}{n}$$

$$= k_x \cdot \frac{1}{n^2} \sum_{i=1}^n B_i + \frac{k_y}{n} D + \frac{n-1}{n} C_i$$

Known from the model, the optimal bidding of bidder i is related to his own cost, his own bidding price, bidding price of competitors, the number of bidders, and base price compiled by the tenderee.

V. COMPARATIVE ANALYSIS OF THESE TWO MODELS

(1) These two models have the same hypothesis premise, so modeling processes are accordant. However, compound base price and without base price have different computing paradigm, so calculation results of optimal bidding are different.

(2) The form of without base price is the international general way. The optimal bidding is related to bidding price of competitors, so bidders can collude together to control bidding price. Law on Public Bidding should stipulate joint bidding strictly to prevent collusion phenomenon. Reference [4] also gives methods in prequalification to prevent this phenomenon.

(3) Compound base price is difficult to operate, but it is only one kind of evidence for bid winning in this bidding process. Hence, bidders must pay attention to technology and business documents to win a bid. In order to improve competitiveness, bidders must strengthen management, increase the credibility, use advanced construction organization mode, and use advanced technology. Because the optimal bidding of bidder i is related to his own cost, his own bidding price, bidding price of competitors, the number of bidders, and base price compiled by the tenderee in compound base price, bidders should analyse bidding documents in detail, understand the situation of competitors fully, and estimate bidding price of competitors accurately.

(4) In addition, due to the superiority of compound base price, this form will be recognized by more and more

tenderees. Thus, bidders should understand way and means of bid evaluation well, which is beneficial to bid decision.

VI. CONCLUSION

(1) Based on game theory and the characteristics in bidding, this paper constructed bidding price game model of without base price and bidding price game model of compound base price.

(2) These two models find relevant factors of optimal bidding. Therefore, bidders can improve bid winning probability by analysing these factors correctly.

(3) This paper gives measures to prevent string bid phenomenon through the comparative analysis of these two models.

(4) This paper advises bidders to understand compound base price.

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